

The investigation

Class Discussion

Explain to the pupils that they are going to tackle a series of investigations. Discuss the problem solving strategies that the pupils are familiar with. Highlight the following as being useful for these investigations.

- Draw a diagram – this is likely to be a very useful strategy. We are going to use arrow diagrams and circle diagrams in this case.
- Look for a pattern – these investigations are all about pattern in numbers. There will be lots of patterns to spot!

Discuss the importance of the pupils recording their thinking *as they work*. They will need to draw/write their diagrams and calculations in a way that allows them to refer back to work they have already done, so it will be helpful to keep their notes organised as they go along.

Open up the pattern builder for 'Adding 2' on the interactive board.

The screenshot shows the 'Pattern Builder' interface for 'Adding 2'. At the top, there are navigation tabs: Ladder, Investigation, Pattern Builder (selected), Skill Check, Pair Game, and Quiz. Below the tabs is a list of operations: Add, Subtract, Add and Subtract, Addition triangle, Number Story, Subtract and Add, and Subtraction Triangle. A vertical list of numbers 1 through 9 is on the right. The main area contains a 'CHOOSE ONE' section with two options: 'Straightforward Questions' and 'Missing Number Problems'. Below this is the 'Pattern Builder - Add a single digit' window. It has a 'Choose the starting number' section with a grid of numbers 1-9 and a 'Fill down' section with a grid of numbers 1-9. The 'Fill down' section shows the equation $? + 2 = ?$ repeated for each number. There are 'Help', 'Check', and 'Start Again' buttons at the bottom.

Ask the pupils for a 2-digit starting number ending in zero (eg 40).

Show them how to build a pattern beginning with 40 and repeatedly adding 2 as follows.
(A double use of the 'fill down' tool makes the pattern very quick to build.)

Pattern Builder - Add a single digit

Well done!

1	2	3
4	5	6
7	8	9
-	0	.

[Help](#) [Check](#) [Start Again](#)

Fill down

$40 + 2 = 42$ ✓

$42 + 2 = 44$ ✓

$44 + 2 = 46$ ✓

$46 + 2 = 48$ ✓

$48 + 2 = 50$ ✓

$50 + 2 = 52$ ✓

Show the pupils how to record these numbers as a number chain:

Add 2

$40 \rightarrow 42 \rightarrow 44 \rightarrow 46 \rightarrow 48 \rightarrow 50 \rightarrow 52$ etc.

Repeat with a different starting number (eg 64) and build a second chain below the first one.

Pattern Builder - Add a single digit

Well done!

1	2	3
4	5	6
7	8	9
-	0	.

[Help](#) [Check](#) [Start Again](#)

Fill down

$64 + 2 = 66$ ✓

$66 + 2 = 68$ ✓

$68 + 2 = 70$ ✓

$70 + 2 = 72$ ✓

$72 + 2 = 74$ ✓

$74 + 2 = 76$ ✓

Continue with several more starting numbers until you have a series of chains like this:

Add 2

$40 \rightarrow 42 \rightarrow 44 \rightarrow 46 \rightarrow 48 \rightarrow 50 \rightarrow 52$

$64 \rightarrow 66 \rightarrow 68 \rightarrow 70 \rightarrow 72 \rightarrow 74 \rightarrow 76$

$28 \rightarrow 30 \rightarrow 32 \rightarrow 34 \rightarrow 36 \rightarrow 38 \rightarrow 40$

$17 \rightarrow 19 \rightarrow 21 \rightarrow 23 \rightarrow 25 \rightarrow 27 \rightarrow 29$

$59 \rightarrow 61 \rightarrow 63 \rightarrow 65 \rightarrow 67 \rightarrow 69 \rightarrow 71$

etc

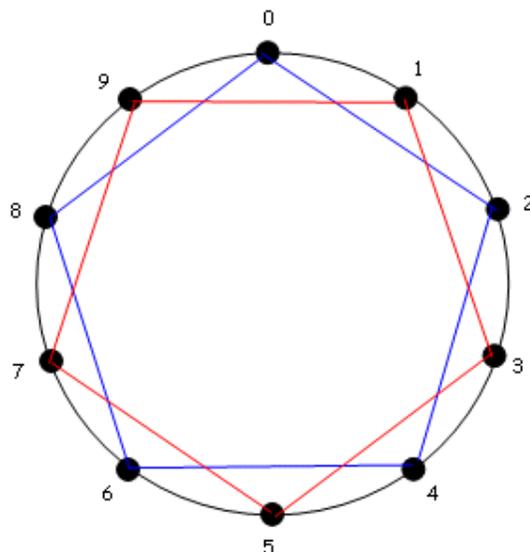
As you work, engage the children in discussion about the patterns. Highlight the following points (you will hopefully get these from the pupils)

- The ending digits go in a pattern that keeps repeating (eg $0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 0 \rightarrow 2 \rightarrow 4 \rightarrow$)
- If you start on an even number you get only even numbers.
- If you start on an odd number you get only odd numbers.
- Whatever number you start on, you get the same repeating digit patterns. You just begin in a different place.

Show the pupils how to record their findings using a digit wheel.

Example: Here is the wheel for 'Add 2'.

The dots are joined in the order $0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 0$ and $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9 \rightarrow 1$.



First Investigation - Add or Subtract 2, 4, 6 or 8

Individual, Pair or Team Work

Set the pupils the challenge of exploring what happens when you add or subtract 2, 4, 6 or 8. A suggested order for exploration might be as follows:

Add 2 (the pattern above)

Subtract 2 (the same pattern in reverse).

Add 4 (a new pattern)

Subtract 4 (the same pattern in reverse).

Add 6 (the same pattern as subtract 4)

Subtract 6 (the same pattern as add 4)

Add 8 (the same pattern as subtract 2)

Subtract 8 (the same pattern as add 2)

Follow-Up Discussion

Engage the pupils in discussion about the patterns. Why do they think they work like this?

What would happen if you did 'Add 0' or 'Subtract 0'. (You would get number chains that go nowhere and a digit wheel with no lines, since adding or subtracting 0 leaves you on the number you start on.)

Project (or draw) large copies of the completed digit wheels on the board and use them to practice 'counting in 2s', 'counting in 4s', 'counting in 6s' and 'counting in 8s' starting on any number, moving a pointer round the digit wheel as you count. Practise counting both forwards and backwards. Use the pattern builder to support with finding the correct tens numbers. Note that the tens number goes up or down one each time you go past zero at the top of the wheel.

Follow-up Activities

Get the pupils to work in pairs counting in twos, fours, sixes or eights round the digit wheel. The pupils can take turns, with one putting appropriate numbers into the pattern builder and the other one counting using the wheel without looking at the computer. The one with the computer uses the pattern builder to check the partner's counting. Then swap.

Set the pupils to do the following investigations on the Learning Ladder as consolidation for homework.

Step 2 – Add Multiples of 2 & Subtract Multiples of 2

Second Investigation - Add or Subtract 5

Individual, Pair or Team Work

Set the pupils the challenge of exploring what happens when you add or subtract 5. A new pattern will emerge.

Follow-Up Discussion

Engage the pupils in discussion about the pattern. Why do they think it works like this? Project (or draw) a large copy of the completed pattern on the board and use it to practice 'counting in 5s', starting on any number, moving a pointer back and forth across the digit wheel as you count. Practise counting both forwards and backwards. Use the pattern builder to support with finding the correct tens numbers. Note that the tens numbers go up or down one each time you go past zero at the top of the wheel.

Follow-up Activities

Get the pupils to work in pairs counting in fives using the digit wheel to support. The pupils can take turns, with one putting appropriate numbers into the pattern builder and the other one counting using the wheel without looking at the computer. The one with the computer uses the pattern builder to check the partner's counting. Then swap.

Set the pupils to do the following investigations on the Learning Ladder as consolidation.

Step 2 – Patterns Adding 5 & Patterns Subtracting 5

Third Investigation - Add or Subtract 1, 3, 7, or 9

Individual, Pair or Team Work

Set the pupils the challenge of exploring what happens when you add 1, 3, 7 or 9. A suggested order for exploration might be as follows:

Add 1 (a new pattern)

Subtract 1 (the same pattern in reverse).

Add 3 (a new pattern)

Subtract 3 (the same pattern in reverse).

Add 7 (the same pattern as subtract 3)

Subtract 7 (the same pattern as add 3)

Add 9 (the same pattern as subtract 1)

Subtract 9 (the same pattern as add 1)

Follow-Up Discussion

Engage the pupils in discussion about the patterns. Why do they think they work like this?
Note the symmetry in the patterns

Adding 0 \leftrightarrow Subtracting 0

Adding 1 \leftrightarrow Subtracting 9

Subtracting 1 \leftrightarrow Adding 9

Adding 2 \leftrightarrow Subtracting 8

Subtracting 2 \leftrightarrow Adding 8

Adding 3 \leftrightarrow Subtracting 7

Subtracting 3 \leftrightarrow Adding 7

Adding 4 \leftrightarrow Subtracting 6

Subtracting 4 \leftrightarrow Adding 6

Adding 5 \leftrightarrow Subtracting 5

Project (or draw) large copies of the completed digit wheels on the board and use them to practice 'counting in 1s', 'counting in 3s', 'counting in 7s' and 'counting in 9s' starting on any number, moving a pointer round the digit wheel as you count. Practise counting both forwards and backwards. Use the pattern builder to support with finding the correct tens numbers. Note that the tens number goes up or down one each time you go past zero at the top of the wheel.

Follow-up Activities

Get the pupils to work in pairs counting in ones, threes, sevens or nines round the digit wheel. They can take turns, with one putting appropriate numbers into the pattern builder and the other counting using the wheel without looking at the computer. The one with the computer uses the pattern builder to check the partner's counting. Then swap.

Let the pupils create large versions of the digit wheels for display. They could overlay different wheels on to one another using different colours. From the discussion about symmetry above, the following would be good combinations:

- one colour for 'add 2 / add 8'. and a different colour for 'add 4 / add 6'
- colour for 'add 1 / add 9'. and a different colour for 'add 3 / add 7'
- 'add 5' could be overlaid onto either of these.

The ambitious could try overlaying all five patterns on to each other, using 5 different colours.

Further Development

This investigation would be a useful jumping off point for the further investigations 'Quick Ways of Adding 9 (and 8)' and 'Quick Ways of Subtracting 9 (and 8)'.

It would also link well with any activity for learning the 'stations' of the multiplication tables (eg Counting Caterpillar), since these same patterns are the ones that appear in the final digits in the tables.

Name: _____ Class: _____ Date: _____

Number Chains and Digit Wheels

The Investigation

If you start on any number and keep adding or subtracting the same (single-digit number) what happens to the pattern in the units digits?

Record your thinking using number chains and digit wheels.

Example - Add 2

40 → 42 → 44 → 46 → 48 → 50 → 52 → 54 → 56 → 58 → 60 →

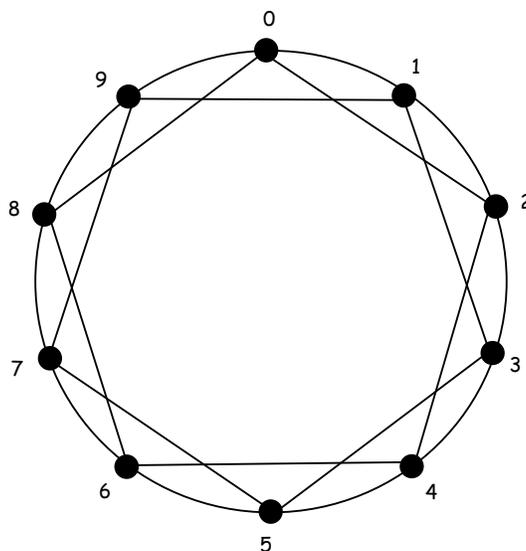
64 → 66 → 68 → 70 → 72 → 74 → 76 → 78 → 80 → 82 → 84 →

17 → 19 → 21 → 23 → 25 → 27 → 29 → 31 → 33 → 35 → 37 →

The units digits either go

0 → 2 → 4 → 6 → 8 → 0 → 2 → 4 → 6 → 8 → 0 →

or 1 → 3 → 5 → 7 → 9 → 1 → 3 → 5 → 7 → 9 → 1 →



Name: _____ Class: _____ Date: _____

Number Chains

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Name: _____ Class: _____ Date: _____

Digit Wheels

